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Quiz one, MTH 221 , Fall 2022

Ayman Badawi

Score = $\frac{15}{15}$

QUESTION 1. (15 points) Let $D = span\{(1,1,0,0), (-1,-1,1,0), (-1,-1,2,0)\}$

7 (i) Find $dim(D)$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & 2 & 0 \end{bmatrix}$$

- kill below
- $1R_1 + R_2 \to R_2$
- $1R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

kill below

$-2R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

independent points

dependent point

Dim (D) = independent points = 2 ✓

3 (ii) Find a basis for D

Basis for D = B = [(1,1,0,0) (0,0,1,0)] $\boxed{\text{or}}$ [(1,1,0,0) (-1,-1,1,0)] ✓

5 (iii) Does $(1,1,2,0) \in D$ yes it's belong $\in D$ ✓

$(1,1,2,0) = c_1(1,1,0,0) + c_2(0,0,1,0)$

$(1,1,2,0) = (c_1, c_1, 0, 0) + (0,0,c_2,0)$

$(1,1,2,0) = (c_1, c_1, c_2, 0)$ $\quad c_1 = 1 \quad c_2 = 2$, $c_1 = 1 \quad 0 = 0$ ✓

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Quiz Two, MTH 221 , Fall– 2022

Ayman Badawi

Score = $\frac{15}{15}$

QUESTION 1. (i) Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ 0 & 4 \\ 1 & 3 \\ 2 & 4 \end{bmatrix}$ Find AB using linear combination of columns method.

5/5

1st column = $2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$

2nd column = $2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 26 \\ 20 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 10 & 26 \\ 10 & 20 \end{bmatrix}$ ✓

(ii) Let $T : R^3 \to R^4$ such that $T(a, b, c) = (0, a, 3b - 2c, -a + c)$

0) Convince me that T is a linear transformation.

a) Write range(T) as span of some points

b) Find all points in the domain of T where $T(a, b, c) = (0, 5, 2, 7)$

c) Find all points in the domain of T where $T(a, b, c) = (0, 0, 0, 0)$

2/2

a) $T(a,b,c) = (0, a, 3b-2c, -a+c)$

$T(0,0,0) = (0,0,0,0) = 0 \Rightarrow$ It may or may not be a linear transformation

$0, a, 3b-2c, -a+c$ are linear combinations of a, b, c

$\Rightarrow T$ is a linear transformation. ✓

3/3

b) Range(T) = $\{(0, a, 3b-2c, -a+c) \mid a,b,c \in R\}$

$= \{a(0,1,0,-1) + b(0,0,3,0) + c(0,0,-2,1)\}$

$= \text{span}\{(0,1,0,-1), (0,0,3,0), (0,0,-2,1)\}$ ✓

c) $T(a,b,c) = (0,5,2,7)$

3/3

$0 = 0$

$a = 5$

$3b - 2c = 2 \Rightarrow 3b = 26 \Rightarrow b = 26/3$

$-a + c = 7 \Rightarrow c = 12$

Soln. set $= \{(5, 26/3, 12)$ ✓

d) $T(a,b,c) = (0,0,0,0)$

2/2

$0 = 0$

$a = 0$

$-a + c = 0 \Rightarrow c = 0$

$3b - 2c = 0 \Rightarrow b = 0$

Soln. set $= \{(0,0,0)\} = \text{span}\{(0,0,0)\}$ ✓

Quiz 3

15/15

Ayda Atallar
87373

$T: \mathbb{R}^2 \to \mathbb{R}$

$T(1,2) = 5$, $T(-1,0) = 3$

1) Find $T(0,2)$

4/4

$T(0,2) = T(1,2) + T(-1,0) = 5 + 3 = 8$

2) Find $T(4(1,2) + -3(-1,0))$

4/4

$4T(1,2) - 3T(-1,0) = 4(5) - 3(3) = 20 - 9 = 11$

$T: \mathbb{R}^3 \to \mathbb{R}$

$T(a,b,c) = a + 6b - c$

1 eq × 3 var ⇒ 1×3 matrix

a) Find all points in the domain of T s.t $T(a,b,c) = 10$

4/4

$$\begin{array}{cccc} a & b & c & \text{const} \\ \end{array}$$
$$\left[\begin{array}{ccc|c} \underline{1} & 6 & -1 & 10 \end{array}\right]$$

$a = 10 - 6b + c$ where a is the leading variable & b, c are free variables

solution = $\{(10 - 6b + c, b, c) \mid b, c \in \mathbb{R}\}$

b) Find all points in the domain of T s.t. $T(a,b,c) = 0$

3/3

$$\begin{array}{cccc} a & b & c & \text{const} \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 6 & -1 & 0 \end{array}\right]$$

$a = -6b + c$

solution = $\{(-6b + c, b, c) \mid b, c \in \mathbb{R}\}$

domain = span$\{(-6,1,0), (1,0,1)\}$

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Quiz 4

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$\frac{15}{15}$

$\frac{5}{5}$ **QUESTION 1.** Let $A = \begin{bmatrix} 1 & 2 & 4 & 6 \\ -1 & -2 & -2 & -2 \\ -2 & 1 & -5 & 10 \\ -1 & -2 & -4 & 4 \end{bmatrix}$

Find $|A|$.

$A \xrightarrow[R_1+R_4 \to R_4]{R_1+R_2 \to R_2,\ 2R_1+R_3 \to R_3} B_1 = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 4 \\ 0 & 5 & 3 & 22 \\ 0 & 0 & 0 & 10 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} B_2 \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 5 & 3 & 22 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

$B_2 \xrightarrow{-3R_2+R_3 \to R_3} B_3 = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 5 & 0 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix} \xrightarrow{R_2 \Leftrightarrow R_3} B_4 = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 5 & 0 & -16 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

$|B_4| = 1 \times 5 \times 1 \times 10 = 50$

$|B_1| = |A|$

$|B_2| = \frac{1}{2}|B_1| = \frac{1}{2}|A|$

$|B_3| = |B_2| = \frac{1}{2}|A|$

$|B_4| = -|B_3| = -\frac{1}{2}|A|$

$50 = -\frac{1}{2}|A|$

$|A| = -100$

$\frac{5}{5}$ **QUESTION 2.** Let $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & -2 & -1 & -1 \\ -1 & -2 & -1 & 0 \end{bmatrix}$ Find a basis for the column space of A.

$\begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & -2 & -1 & -1 \\ -1 & -2 & -1 & 0 \end{bmatrix} \xrightarrow[R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Basis for Col(A) = $\{(1,-1,-1), (1,-1,0)\}$

$\frac{5}{5}$ **QUESTION 3.** Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix}$. Find all eigenvalues of A.

$C_A(\alpha) = |\alpha I_3 - A|$

$\left| \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix} \right| = \begin{vmatrix} \alpha & 0 & 0 \\ 0 & \alpha-1 & -4 \\ -1 & 0 & \alpha \end{vmatrix} = C_A(\alpha)$

$\alpha(\alpha(\alpha-1)) = \alpha(\alpha^2-\alpha) = \alpha^3-\alpha^2 = C_A(\alpha)$

$C_A(\alpha) = 0$

$\alpha^3 - \alpha^2 = 0$

$\alpha^2(\alpha-1) = 0$

$\alpha = 1, 0$

$\therefore$ The eigenvalues of A are 1 and 0

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Quiz 5

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15/15

QUESTION 1. i) Let $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix}$

Find A^{-1} if possible, then find $(A^T)^{-1}$. $= (A^{-1})^T$

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1+R_2 \to R_2 \\ R_1+R_3 \to R_3 \\ R_1+R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} -2R_4+R_3 \to R_3 \\ R_1 \Leftrightarrow R_2 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -R_3+R_2 \to R_2 \\ -R_4+R_2 \to R_2 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ -1 & 0 & 1 & -2 \\ 1 & 0 & 0 & 1 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

9/9

ii) Find the solution set of the following system

$$A^{-1} \times A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = A^{-1} \times \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ -1 & 0 & 1 & -2 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -5 \\ 2 \end{bmatrix}$$

Sol. set $= \{(2, 4, -5, 2)\}$

3/3

QUESTION 2. Let $A = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$ Find A^{-1}.

$|A| = (2 \times 3) - (2 \times 2)$
$= 6 - 4 = 2$
so A^{-1} exists.

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix}$$

3/3

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Quiz 6

15/15

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QUESTION 1. Let $T : R^3 \to R^2$ be a linear transformation such that $T(a, b, c) = (a + 2b + c,\ -2a - 4b - c)$

1) Find a basis for the Range(T). Is T ONTO?

$M_T = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -1 \end{bmatrix} \xrightarrow{2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ basis for Range(T) = $\{(1,-2), (1,-1)\}$ ✓

dim (Range(T)) = 2 = dim (codomain (T)) ✓ 3/3

∴ T is onto

2)Find a basis for the Ker(T). Is T one-to-one?

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -2 & 4 & -1 & 0 \end{array}\right] \xrightarrow{2R_1 + R_2 \to R_2} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array}$ $c = 0$

$a + 2b + c = 0 \Rightarrow a = -2b$ 3/3

basis for Ker(T) = $\{(-2, 1, 0)\}$ ✓ Ker(T) ≠ Origin ∴ T is not one to one ✓

QUESTION 2. Let $T : R^3 \to R^2$ and $L : R^2 \to R^3$ be linear transformations such that $T(a, b, c) = (a + b, c)$ and $L(a, b) = (0, a, a + b ~~+ c~~)$. Then we know $L\ o\ T : R^3 \to R^3$ is a linear transformation. Find the standard matrix presentation of $L\ o\ T$.

$M_{LoT} = M_L M_T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ ✓ ✓

4/4

QUESTION 3. Given A is a 3×3 matrix such that 2, 4, 1 are the eigenvalues of A. Let $B = A^2 - 4A^{-1} - 2I_3$.

1) Find all eigenvalues of B.

let eigenvalues of B be B_1, B_2, B_3

$B_1 = 2^2 - \frac{4}{2} - 2 = 0$ ✓ 3/3

$B_2 = 4^2 - \frac{4}{4} - 2 = 13$ ✓

$B_3 = 1^2 - 4 - 2 = -5$ ✓ ∴ Eigenvalues = 0, 13, -5

2) Convince me that A is invertible, but B is not invertible.

$|A| = 2 \cdot 4 \cdot 1 = 8 \neq 0 \Rightarrow$ A is invertible ✓ 2/2

$|B| = 0 \cdot 13 \cdot -5 = 0 \Rightarrow$ B is not invertible ✓

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Quiz 7

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15/15

QUESTION 1. Let $T : P_2 \to P_2$ be a linear transformation such that $T(ax + b) = (a + 3b)x + b$

1) Find all eigenvalues of T

$\cancel{L(a,b) = (3b\,,\ a+b)}$

$\cancel{C_{M_L}(\alpha) = |I_2\alpha - M_L| = |}$

$L(a,b) = (a+3b\,,\ b)$

$C_{M_L}(\alpha) = |I_2\alpha - M_L| = \begin{vmatrix} \alpha - 1 & -3 \\ 0 & \alpha - 1 \end{vmatrix}$

$= (\alpha - 1)^2$ ✓ 3/3

$C_{M_L}(\alpha) = 0 \Rightarrow \alpha = 1$ is the eigenvalue for T

2) For each eigenvalue, α, find the eigenspace $E_\alpha(T)$.

$E_1 : \left[\begin{array}{cc|c} 0 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $a = a\,,\ b = 0$ $\quad E_1 = \{(a,0) \mid a \in \mathbb{R}\} \to$ for L

E_1 for T $= \{ax \mid a \in \mathbb{R}\} = \text{span}\{x\}$ ✓ 2/2

QUESTION 2. Given $D = \Big\{f(x) \in P_3 \mid f(-1) = 0\Big\}$ is a subspace of P_3. Find a basis for D.

$f(-1) = 0 \Rightarrow f(x) = ax^2 + bx + c \quad f(-1) = a - b + c = 0 \Rightarrow a = b - c$

$D = \{(b-c)x^2 + bx + c\} = \text{span}\{(x^2+x), (-x^2+1)\}$ ✓

$x^2 + x$ and $-x^2 + 1$ are independent

$\therefore$ basis for $D = \{(x^2+x), (-x^2+1)\}$ ✓ 4/4

QUESTION 3. Let $T : R^{2\times 2} \to P_3$ such that $T(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = (a+b+c+2d)x^2 + (-a-b-c-d)x + -d$

1) Find a basis for the $Range(T)$

$L(a,b,c,d) = (a+b+c+2d\,,\ -a-b-c-d\,,\ -d)$

$M_L = \begin{bmatrix} 1 & 1 & 1 & 2 \\ -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\text{Range}(L) = \text{span}\{(1,-1,0), (2,-1,-1)\} \therefore \text{Range}(T) = \text{span}\{(x^2 - x), (2x^2 - x - 1)\}$ ✓ 5/5

2) Is T ONTO? explain.

$\dim(\text{Range}(T)) = 2$

$\dim(\text{codomain}(T)) = 3$

$\dim(\text{Range}(T)) \neq \dim(\text{codomain}(T)) \quad \therefore T$ is not onto ✓ 1/1

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Quiz 8 MTH–221 , Fall 2022

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Score = $\frac{\ \ }{15}$

15/15

QUESTION 1. Let A be a 3×3 matrix such that $C_A(\alpha) = (\alpha-2)^2(\alpha+3)$. Given, $E_2(A) = span\{(1,0,-3),(-1,0,4)\}$ and $E_{-3}(A) = span\{(-1,2,3)\}$.

a)Is A diagnolizable ? explain. If yes, find a diagonal matrix D and invertible matrix Q such that $Q^{-1}AQ = D$.

dim ($\cancel{E_2}$ span $(E_2(A))) = 2$ = no. of times 2 is repeated

dim (span $(E_{-3}(A))) = 1$ $\therefore$ A is diagnolizable

$$D = \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}}_{2\ 2}, \quad Q = \underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 2 \\ -3 & 4 & 3 \end{bmatrix}}_{2\ 2}$$

5/5

QUESTION 2. Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$. Is A diagnolizable ? explain. If yes, find a diagonal matrix D and invertible matrix Q such that $Q^{-1}AQ = D$.

$$C_A(\alpha) = \begin{vmatrix} \alpha & 1 \\ -1 & \alpha-2 \end{vmatrix} = \alpha(\alpha-2)+1 = (\alpha-1)^2$$

$\alpha = 1$ is the eigenvalue of A

$$E_1(\alpha) = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & -1 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

5/5

no. of free variables = 1

$\therefore$ dim $(E_1(\alpha)) = 1 \neq 2$ (no. of times 1 is repeated)

$\therefore$ A is not diagnolizable.

QUESTION 3. Let $A = \begin{bmatrix} 0 & -6 \\ 1 & 5 \end{bmatrix}$. Is A diagnolizable ? explain. If yes, find a diagonal matrix D and invertible matrix Q such that $Q^{-1}AQ = D$.

$C_A(\alpha) = \begin{bmatrix} \alpha & 6 \\ -1 & \alpha - 5 \end{bmatrix} = \alpha(\alpha-5) + 6 = (\alpha-3)(\alpha-2)$

$\alpha = 2, 3$ are the ~~ratio~~ eigenvalues and they are only repeated once

$\therefore$ A is diagnolizable.

$D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ ✓ X

$E_2(\alpha)$: $\left[\begin{array}{cc|c} 2 & 6 & 0 \\ -1 & -3 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 2 & 6 & 0 \\ 0 & 0 & 0 \end{array}\right]$ leading var. = a
free var. = b
a = - 3b

1.5/1.5

$E_2(\alpha) =$ span$\{(-3, 1)\}$ ✓

$E_3(\alpha)$: $\left[\begin{array}{cc|c} 3 & 6 & 0 \\ -1 & -2 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 3 & 6 & 0 \\ 0 & 0 & 0 \end{array}\right]$ a = -2b

1.5/1.5

$E_3(\alpha) =$ span$\{(-2, 1)\}$ ✓

$Q = \begin{bmatrix} -3 & -2 \\ 1 & 1 \end{bmatrix}$ ✓ X

5/5

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MTH 221 , Fall 2022, 1–1

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$\frac{15}{15}$

Quiz 9

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$\frac{3}{3}$ **QUESTION 1.** (a) Let $A = \begin{bmatrix} 2 & 2 \\ -1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$. Find $A \otimes B$.

$$A \otimes B = \begin{bmatrix} 2\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} & 2\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \\ -1\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} & -3\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 6 & -2 & 6 & -2 \\ 4 & 2 & 4 & 2 \\ -3 & 1 & -9 & 3 \\ -2 & -1 & -6 & -3 \end{bmatrix}$$

(b) Given A, B are 2×2 matrices such that $2, 1$ are the eigenvalues of A and $2, -1$ are the eigenvalues of B.

$\frac{3}{3}$ i) Find all eigenvalues of $A \otimes B$.

$(2,1) \Rightarrow$ eigen of A

$(2,-1) \Rightarrow$ eigen of B

$(2,2) \Rightarrow 4$

$(2,-1) \rightarrow -2$

$(1,2) \Rightarrow 2$

$(1,-1) \Rightarrow -1$

$\therefore$ eigenValues of $A \otimes B = 4, -2, 2, -1$

$\frac{3}{3}$ ii) Find $|A \otimes B|$

$= 4 \times -2 \times 2 \times -1 = 16$

$\frac{3}{3}$ **QUESTION 2.** (i) Use the integral inner product on P_3 where $a = 0$ and $b = 1$. Find the distance between $f_1(x) = 4x + 2$ and $f_2(x) = x + 2$.

$$|f_1 - f_2| = |(4x+2) - (x+2)| = |3x+0| = \sqrt{\langle 3x+0, 3x+0 \rangle}$$

$$= \sqrt{\int_0^1 (3x+0)^2 dx} = \sqrt{\int_0^1 (3x)^2 dx} = \sqrt{3}$$

$\frac{3}{3}$ ii) Use the normal dot product on R^2. Find the angle between $Q_1 = (3,4)$ and $Q_2 = (-3,4)$

$$\cos(\theta) = \frac{\langle Q_1, Q_2 \rangle}{|Q_1|\,|Q_2|} = \frac{\langle (3,4), (-3,4) \rangle}{|3,4|\,|-3,4|} = \frac{3(-3) + 4(4)}{\sqrt{3(3)+4(4)}\,\sqrt{-3(-3)+4(4)}}$$

$$= \frac{7}{\sqrt{25} \times \sqrt{25}} = \frac{7}{25}$$

$\therefore \theta = \cos^{-1}\left(\frac{7}{25}\right) = 73.73979529^\circ$

$= 73.74^\circ$

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